

An interesting fact that I noted was, if the complex numbers  $a+bi$  and  $x+yi$  were denoted as vectors,  $\begin{pmatrix} a \\ b \end{pmatrix}$  and  $\begin{pmatrix} x \\ y \end{pmatrix}$ , then

$$\begin{pmatrix} x \\ y \end{pmatrix} \div \begin{pmatrix} a \\ b \end{pmatrix} = \left( \begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ -b \end{pmatrix} \cdot \begin{pmatrix} y \\ x \end{pmatrix} i \right) \times \frac{1}{a^2+b^2}$$

This representation of  $\frac{x+yi}{a+bi}$  could be used to construct it.

Draw the position vectors of the points  $(a, b)$  and  $(x, y)$ ,  $\begin{pmatrix} a \\ b \end{pmatrix}$  and  $\begin{pmatrix} x \\ y \end{pmatrix}$ . This construction is for the case where  $\arg(x+yi) > \arg(a+bi)$ . {If  $\arg(a+bi) > \arg(x+yi)$  then swap your vectors around.}

Measure the lengths of the vectors. Construct the perpendicular from the point  $\begin{pmatrix} a \\ b \end{pmatrix}$  to the vector  $\begin{pmatrix} x \\ y \end{pmatrix}$  {or its extension}. {Let the point where this perpendicular meets  $\begin{pmatrix} x \\ y \end{pmatrix}$  or its extension have position vector  $k$ }. This is the projection of  $\begin{pmatrix} a \\ b \end{pmatrix}$  onto  $\begin{pmatrix} x \\ y \end{pmatrix}$ .

Measure the length of the position vector  $k$ . Scale  $k$  up by the factor of the length of  $\begin{pmatrix} x \\ y \end{pmatrix}$ . The length of this new vector is  $\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$ .

Rotate this vector so that it lies on the real axis. To do this, draw a circle with its radius as the position vector  $k$ , centered at the origin and find the position vector of the point where the circle crosses the positive real axis. This new vector is the position vector  $c$  of the point  $(ax+by, 0)$ .

Now, reflect the point  $(x, y)$  in the line  $y=x$  and draw the position vector  $\begin{pmatrix} y \\ x \end{pmatrix}$  of the point  $(y, x)$ . Reflect the point  $(a, b)$  in the real axis and draw the position vector  $\begin{pmatrix} a \\ -b \end{pmatrix}$  of the point  $(a, -b)$ . Construct the perpendicular from the point  $(a, -b)$  to  $\begin{pmatrix} y \\ x \end{pmatrix}$ .

Measure the length of the position vector  $m$ , the projection of  $\begin{pmatrix} a \\ -b \end{pmatrix}$  onto  $\begin{pmatrix} y \\ x \end{pmatrix}$ . Scale  $m$  up by a factor equal to the length of  $\begin{pmatrix} y \\ x \end{pmatrix}$  (which of course is equal to the length of  $\begin{pmatrix} x \\ y \end{pmatrix}$ ). The length of this new vector is  $\begin{pmatrix} a \\ -b \end{pmatrix} \cdot \begin{pmatrix} y \\ x \end{pmatrix}$ . Now, rotate this vector so that it lies on the imaginary axis using the same method described above. This new vector is the position vector  $d$  of the point  $(0, ay-bx)$ . Now draw the vector  $c+d$  using the parallelogram method (in this case, the parallelogram is a rectangle). Scale this vector down by a factor equal to the square of the length of the vector  $\begin{pmatrix} a \\ b \end{pmatrix}$ . This is the position vector of the complex

number  $\frac{x+yi}{a+bi}$