

An interesting fact that I noted was, if the complex numbers $a+bi$ and $x+yi$ were denoted as vectors, $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$, then

$$\begin{pmatrix} x \\ y \end{pmatrix} \div \begin{pmatrix} a \\ b \end{pmatrix} = \left(\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ -b \end{pmatrix} \cdot \begin{pmatrix} y \\ x \end{pmatrix} i \right) \times \frac{1}{a^2 + b^2}$$

This representation of $\frac{x+yi}{a+bi}$ could be used to construct it.

Draw the position vectors of the points (a,b) and (x,y) , $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} x \\ y \end{pmatrix}$. This construction is for the case where $\arg(x+yi) > \arg(a+bi)$. {If $\arg(a+bi) > \arg(x+yi)$ then swap your vectors around.}

Measure the lengths of the vectors. Construct the perpendicular from the point (a,b) to the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ {or its extension}. {Let the point where this perpendicular meets $\begin{pmatrix} x \\ y \end{pmatrix}$ or its extension have position vector \underline{k} }. This is the projection of $\begin{pmatrix} a \\ b \end{pmatrix}$ onto $\begin{pmatrix} x \\ y \end{pmatrix}$.

Measure the length of the position vector \underline{k} . Scale \underline{k} up by the factor of the length of $\begin{pmatrix} x \\ y \end{pmatrix}$. The length of this new vector is $\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$.

Rotate this vector so that it lies on the real axis. To do this, draw a circle with it's radius as the position vector \underline{k} , centered at the origin and find the position vector of the point where the circle crosses the positive real axis. This new vector is the position vector \underline{c} of the point $(ax+by, 0)$.

Now, reflect the point (x,y) in the line $y=x$ and draw the position vector $\begin{pmatrix} y \\ x \end{pmatrix}$ of the point (y,x) . Reflect the point (a,b) in the real axis and draw the position vector $\begin{pmatrix} a \\ -b \end{pmatrix}$ of the point $(a,-b)$. Construct the perpendicular from the point $(a,-b)$ to $\begin{pmatrix} y \\ x \end{pmatrix}$.

Measure the length of the position vector \underline{m} , the projection of $\begin{pmatrix} a \\ -b \end{pmatrix}$ onto $\begin{pmatrix} y \\ x \end{pmatrix}$. Scale

\underline{m} up by a factor equal to the length of $\begin{pmatrix} y \\ x \end{pmatrix}$ (which of course is equal to the length of $\begin{pmatrix} x \\ y \end{pmatrix}$). The length of this new vector is $\begin{pmatrix} a \\ -b \end{pmatrix} \cdot \begin{pmatrix} y \\ x \end{pmatrix}$. Now, rotate this vector so that it lies on the imaginary axis using the same method described above. This new vector is the position vector \underline{d} of the point $(0, ay-bx)$. Now draw the vector $\underline{c} + \underline{d}$ using the parallelogram method (in this case, the parallelogram is a rectangle). Scale this vector down by a factor equal to the square of the length of the vector $\begin{pmatrix} a \\ b \end{pmatrix}$. This is the position vector of the complex

number $\frac{x+yi}{a+bi}$